Stability and Sensitivity for Congestion Control in Wireless Networks with Time Varying Link Capacities

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Abstract

While extensive efforts have been devoted to providing optimization based, distributed congestion control schemes for efficient bandwidth utilization and fair allocation in both wireline and wireless networks, a common assumption therein is fixed link capacities. This unfortunately will limit the application scope in multi-hop wireless networks where channels are ever changing. In this paper, we explicitly model link capacities to be time varying and investigate congestion control problems in multi-hop wireless networks. In particular we propose a primal-dual congestion control algorithm which is proved to be trajectory stable in the absence of feedback delay. Different from system stability around a single equilibrium point, trajectory stability guarantees the system is stable around a time varying reference trajectory. Moreover, we obtain sufficient conditions for the scheme to be locally stable in the presence of delay. Our key technique is to model time variations of capacities as perturbations to a constant link. Furthermore, to study the robustness of the algorithm against capacity variations, we investigate the sensitivity of the control scheme and through simulations to study the tradeoff between stability and sensitivity.

1 Introduction

The inadequacy of TCP when facing the exploding Internet bandwidth has promoted extensive research toward new congestion control schemes targeting at high utilization, low queueing delay, and fairness [23]. Owing to the seminal work by Kelly [9], congestion control has mainly been formulated as utility maximization problems and distributed control theory based solutions have been devised accordingly [6–8, 16, 18, 23]. The proposed schemes generally consist two components: a source algorithm that adjusts sending rate in response to congestion in its path, and a link algorithm that updates a congestion measure and feeds it back, implicitly or explicitly, to the sources utilizing the link.

In parallel, intensive research efforts have been devoted to designing congestion control algorithms capable of accommodating error-prone and time varying wireless links, which have been demonstrated to be well beyond normal TCP's reach [5]. While earlier approaches, such as I-TCP and Snoop-TCP [4], have mainly been engineered based on empirical techniques, recent efforts have embraced the above optimization and control theory based approach [1, 14, 20, 24]. For example, a hop-by-hop congestion control scheme is proposed in [24] specifically for wireless networks and various performance metrics to be maximized in wireless network are studied in [20].

Surprisingly, these congestion schemes developed for wireless networks often have assumed constant link capacities (or a fixed portion of certain constant bandwidth). While such assumptions suit wireline networks comfortably, it certainly will limit the application scopes of the proposed algorithms in the wireless domain. It is well known that wireless channels are characterized by inherent time-varying capacities [21] that have been shown to significantly reduce the throughput of TCP [4]. The failure of TCP in such a scenario is the consequence of its inability to distinguish packet loss caused by flow congestion or link errors (or equivalently, reduced link bandwidth). Using constant capacities to model wireless links cannot fully capture this effect and in particular that on congestion control algorithms. While an optimal congestion control scheme in conjunction with power control has been developed in [1] for multi-hop wireless network considering time-varying link capacities, the requirement of knowledge of network-wise interference limits its practicability.

In this paper, we investigate congestion control in multi-hop wireless networks exemplified by wireless mesh networks with explicitly modeled time varying links. We propose a primal-dual congestion control algorithm and prove it to be trajectory stable in the absence of feedback delay. Different from existing works that can only guarantee system equilibrium at a single point, we show that the system is stable around a sequence of time-indexed equilibrium points that in turn jointly form a time varying reference trajectory. Moreover, by modeling capacity variation as perturbation to a fixed channel, we further obtain sufficient conditions for the primal-dual scheme to be locally stable. Furthermore, to study the robustness of the algorithm against capacity variations, we investigate the sensitivity of the control scheme. Using tractable scenarios, we demonstrate that local stability and system sensitivity in the presence of feedback delay can achieve a balancing tradeoff by tuning the gain of source controllers. Through extensive experimental studies, we show that the algorithm excels in a wide variety of system setups and investigate the effects of different parameters on system stability and sensitivity.

The reminder of this paper is organized as follows. In Section 2, we further motivate our work and present a primal-dual congestion control algorithm. In Section 3, we prove that in the absence of feedback delay, the algorithm is trajectory stable. When feedback delay is present, we derive in Section 4 sufficient conditions for the system to be locally stable. Subsequently, system sensitivity with respect to link capacity perturbation is analyzed in Section 5. Section 6 describes our experimental studies, followed by detailed discussion of related work in Section 7. Finally, we conclude in Section 8.

2 Problem Formulation and Algorithm Design

2.1 Preliminary and Motivation

Triggered by the seminal work by Kelly, congestion control has then been developed and analyzed as distributed algorithms solving appropriately formulated utility maximization problems [9]. Intuitively, consider a wireline communication network with L links, each with a fixed capacity of c_l, and S sources with transmission rate of x_s for $s \in S$. Assume that each source s uses a fixed set $L(s)$ of links to route its traffic through and possesses an increasing, strictly concave, and twice differential utility function $U_s(x_s)$. A congestion control scheme can be formulated as to maximize the total utility $\sum_{s\in S} U_s(x_s)$ over the source rates $\{x_s, s \in S\}$, subject to the constraints of total link capacity, i.e., $\sum_{s:l\in L(s)} x_s \leq c_l$ for all links. Formally, the optimization problem is

$$\max \quad \sum_{s\in S} U_s(x_s) \tag{1}$$

$$s.t. \quad \sum_{s:l\in L(s)} x_s \leq c_l, \forall\, l \in L \tag{2}$$

$$x_s \geq 0, s \in S. \tag{3}$$

Corresponding to the above objective, distributed solutions allowing individual sources to adjust their transmitting rates generally have taken the following form:

$$\dot{x}_s(t) = f\left(x_s(t), \sum_{l\in L(s)} p_l(t)\right) \tag{4}$$

$$\dot{p}_l(t) = g\left(p_l(t), \sum_{s\in S(l)} x_s(t)\right). \tag{5}$$

Here $p_l(t)$ denotes the price of link l that may correspond to link congestion, loss probability, etc. $f(\cdot)$ and $g(\cdot)$ are the control functions for updating the transmission rate and price at the source and link respectively. A unique equilibrium can be derived as the solution for the utility maximization problem for Equations (4) and (5) under certain conditions. Actually the above control based solutions are termed primal-dual algorithm, as differential equations are used at both the sources and links for updating. On the contrary, if static function is employed at the links to generate congestion signal, it is termed primal algorithm and if the sources use static functions to regulate packet rate, it is termed dual algorithms.

Following this approach, while extensive work [7, 9, 11, 12, 15–18, 23] has been done on congestion control thereafter, a common assumption is that the capacity of a link is fixed, i.e., in the optimization problem c_l is not time varying. While for wireline networks, this assumption surely is valid, wireless links, on the contrary, are characterized by time variations. Such variations are direct results of changes in signal to noise ratio (SNR), which in turn are caused by the mobility of wireless devices and/or fluctuations of the surrounding physical environment [21]. While adaptive modulation and coding schemes can be employed at the physical layer, they only target at a desired bit/frame error probability at the cost of varying transmission rate. Indeed, even for wireline networks, bandwidth can also vary due to various reasons. For example, due to link sharing, a router may only have access to a portion of the bandwidth which can fluctuate over time [25]. Regardless, in a plethora of schemes, link capacity c_l shall be replaced by $c_l(t)$. Evidently, time varying capacities demand new investigation on the stability and optimality of traffic congestion control algorithms.

Before formally formulating our problem, we clarify that we assume that $c_l(t)$'s are independent of each other. In multi-hop wireless networks, while flows routed through a common relaying node are subject to resource competition and wireless transmissions are constantly interfering with each other, we assume that techniques based on, for example, spread spectrum technologies (code division multiple access or frequency hopping) can effectively alleviate this problem. Our focus is thus given to end-to-end congestion control schemes that can effectively handle link capacity variations due to the ever-changing channel conditions.

2.2 Problem Formulation

For completeness, we rephrase congestion control for utility maximization but in an all wireless network (wireline links with fixed capacities can be considered a special case). Given the set of wireless links L and the set of traffic sources $\mathcal{S}$. Each source $s \in \mathcal{S}$ identifies a unique source-destination pair and correspondingly a flow between them. Associated with each source is a route r composed of a subset $\{l\} \subset L$ of links. If route r uses link l, we write $l \in r$. Let R be the set of routes. The routing matrix $\mathcal{R}$, of dimension $|L| \times \mathcal{S}$, is thus defined by

$$\mathcal{R}_{lr} = \begin{cases} 1 & \text{if route } r \text{ uses link } l \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Let $x_r \geq 0$ be the flow (source sending) rate associated with route $r \in R$ and $c_l(t)$ be the *time-varying "capacity"* of link $l \in L$. As each link l may be used by several routes, let y_l be the total arrival rate of traffic on logical link l. Then, the vector of link rates $\mathbf{y}$ is given by the relationship $\mathbf{y} = \mathcal{R}\mathbf{x}$, where $\mathbf{y} = (y_l, l \in L)$ and source rate $\mathbf{x} = (x_r, r \in R)$ are both column vectors. For the utility function associated with each source, we restrict ourselves to weighted proportionally fair utility function of the form $U_r(\cdot) = w_r \log(\cdot)$, where w_r is the weight for flow r. This function has been shown to be particularly suitable for wireless networks [20]. The congestion control problem for utility maximization can then be summarized as

$$\text{Max} \quad \sum_{r \in R} w_r \log x_r \tag{7}$$

$$s.t. \quad \sum_{r: l \in r} x_r \leq c_l(t), \quad \forall\, l \in L, x_r \geq 0, r \in R. \tag{8}$$

Again, the key difference in our problem formulation lies in the right side of Equation (8), namely the time varying channel capacity. Correspondingly, the congestion control algorithm must be capable of accommodating the fluctuations while maintaining system stability and optimality. Towards this end, we define a capable primal-dual algorithm below.

2.3 Congestion Control Algorithm

Define Lagrangian function

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\lambda}) &= \sum_{r \in R} w_r \log x_r - \sum_{l \in L} \lambda_l \left(\sum_{r: l \in r} x_r - c_l(t) \right) \\ &= \sum_{r \in R} \left(w_r \log x_r - x_r \sum_{l \in r} \lambda_l \right) + \sum_{l \in L} \lambda_l c_l(t) \end{aligned} \tag{9}$$

where $\lambda_l (l \in L)$ is Lagrangian multiplier. By differentiating Equation (9) with respect to x_r, we have

$$\frac{\partial L}{\partial x_r} = \frac{w_r}{x_r} - \sum_{l \in r} \lambda_l = 0 \tag{10}$$

From the above equation, we get

$$x_r = \frac{w_r}{\sum_{l \in r} \lambda_l} \tag{11}$$

We remark that in Equations (9)-(11), if t is given, then $c_l(t)$ is fixed and bounded. The problem will degenerate to the one presented in [9] and solutions proposed therein hence can be employed. However, as we will show later, the time-varying characteristic of the link capacity will challenge us to explore new techniques for stability proof and furthermore, sensitivity study.

Let $x_r(t)$ denote the flow rate of route r at time t. We define the source rate controller (primal algorithm) that adapts its rate according to the following differential equation

$$\dot{x}_r(t) = k_r \left(w_r - x_r(t) \sum_{l \in r} \lambda_l \right), \tag{12}$$

where λ_l, the Lagrangian multiplier, can also be considered as the link price of link l. Although Equation (12) is analogous in shape to those developed for wireline networks [9], the key difference dwells in λ_l, which now is not only determined by the aggregate rate $y_l(t)$ on link l, but also affected by the variations of link capacity $c_l(t)$. This is further elaborated by the dual algorithm for price updating on each link given by [23]

$$\dot{\lambda}_l(t) = h_l(\lambda_l(t)) \left[y_l(t) - c_l(t) \right]^+_{\lambda_l}. \tag{13}$$

Here, $h_l(\lambda_l(t))$ is a non-decreasing continuous function in a generic form used for price updating. Specific functions can be determined for different purposes. $[y_l(t) - c_l(t)]^+_{\lambda_l}$ is defined as

$$[y_l(t) - c_l(t)]^+_{\lambda_l} = \begin{cases} y_l(t) - c_l(t) & \text{if } \lambda_l > 0, \\ \max(y_l(t) - c_l(t), 0) & \text{if } \lambda_l = 0. \end{cases}$$

Before proceeding to the analysis of this congestion control scheme, we recapitulate our motivation. If t is fixed, $c_l(t)$ is a constant and our model can be reduced to the standard model as stated in (1), which has been shown to possess a unique optimum to the optimization problem. However, since $c_l(t)$ is time-varying, the optimum to (7) is not unique. Instead, the optimum is time varying as well. Our key objective in the remainder of the paper is thus to prove the stability and optimality of the above proposed congestion algorithm, even when coping with time varying link capacities.

3 Stability Analysis without Delay

If link capacities are constant, the unique system equilibrium resides on a single point, which indeed is guaranteed by extensive designs [16, 23]. On the contrary, if the link capacities are time varying, the equilibrium of the system becomes dependent on time t. In other words, the equilibrium $\bar{x}_r(t)$ of the system is a curve rather than a point, as it is varying with t. In this section, we adopt the concept of trajectory stability to investigate this time varying system stability without considering propagation time delay, and show that our proposed congestion control scheme is trajectory stable.

The concept of trajectory stability and its difference from equilibrium point stability can be depicted as follows. Consider a time-invariant system $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t)$ with an equilibrium state $\mathbf{x}_e$. $\mathbf{x}_e$ is said to be *equilibrium-point stable* if for any given t_0 and any real $\varepsilon > 0$, there exists a real $\delta(\varepsilon, t_0)$ such that $\|\mathbf{x_0} - \mathbf{x}_e\| < \delta$ implies $\|\mathbf{x}(t) - \mathbf{x}_e\| < \varepsilon$, where $\mathbf{x}_0$ is an initial value. Under this scenario, *global asymptotic stability* denotes that $\mathbf{x}_e$ is a unique equilibrium point which is stable such that every trajectory for $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t)$ converges to it. In contrast, in trajectory stability, a reference trajectory $\mathbf{x}^*(t)$ is employed instead of a single point $\mathbf{x}_e$. $\mathbf{x}(t; \mathbf{x}_0^*, t_0)$ is deemed trajectory stable if for all t_0 and $\varepsilon > 0$, there is a $\delta(\varepsilon, t_0) > 0$ such that $\|\mathbf{x}(t; \mathbf{x}_0, t_0) - \mathbf{x}^*(t; \mathbf{x}_0^*, t_0)\| < \varepsilon$ for all $t \geq t_0$ if $\|\mathbf{x}_0 - \mathbf{x}_0^*\| < \delta$. $\mathbf{x}^*(t)$ is said to be asymptotically stable if it is stable and convergent. Interested readers are referred to [19] for detailed discussions on trajectory stability.

Employing this concept, when the system delay is zero, we have the following theorem.

Theorem 1 *The proposed primal-dual algorithm is asymptotically stable in trajectory-stability.*

Proof : Denote the optimal transmission rate of user r as $\bar{x}_r(t)$. Let $x_r(t)$ be any other rate that satisfies the constraints in (7) at time t. Let $\bar{\lambda}_l(t)$ and $\lambda_l(t)$ be the corresponding link prices for $\sum_{r:l\in r} \bar{x}_r(t)$ and $\sum_{r:l\in r} x_r(t)$ respectively. Let $\mathbf{x}(t) = (x_r(t), r \in R)$. For simplification, define $q_r(t) = \sum_{l\in r} \lambda_l(t)$, i.e., $q_r(t)$ is the sum of the prices of all links on route r at time t. Let $\mathbf{q}(t) = (q_r(t), r \in R)$, and $\boldsymbol{\lambda}(t) = (\lambda_l(t), l \in L)$. From the definition of routing matrix, we have $\mathbf{q}(t) = \mathcal{R}^T \boldsymbol{\lambda}(t)$, where T stands for the transpose of a matrix. As routing matrix $\mathcal{R}$ is usually required to be of full row rank, given $\mathbf{q}(t)$, there exists a unique $\boldsymbol{\lambda}(t)$ such that $\mathbf{q}(t) = \mathcal{R}^T \boldsymbol{\lambda}(t)$.

Similar to [23], we construct the following Lyapunov function,

$$V(\mathbf{x}(t), \boldsymbol{\lambda}(t); t) = \sum_{r\in R} \int_{\bar{x}_r(t)}^{x_r(t)} \frac{1}{k_r \sigma}(\sigma - \bar{x}_r(t)) d\sigma + \sum_{l\in L} \int_{\bar{\lambda}_l(t)}^{\lambda_l(t)} \frac{1}{h_l(\beta)}(\beta - \bar{\lambda}_l(t)) d\beta \tag{14}$$

Notice that different from [23], the integral scope is time dependent in our case. Taking derivative on both sides of Equation (14), we have Equation (15) and subsequently (16) and (17). In (17), the first term $\sum_{r\in R}(x_r(t) - \bar{x}_r(t))\left(\frac{w_r}{x_r(t)} - \bar{q}_r\right) \leq 0$ since $\frac{w_r}{x_r(t)} \downarrow$ as $x_r(t) \uparrow$. The second term $\sum_{l\in L}(\lambda_l(t) - \bar{\lambda}_l(t))(\bar{y}_l(t) - c_l(t)) \leq 0$ as $\bar{\lambda}_l(t) = 0$ if $\bar{y}_l(t) < c_l(t)$. Therefore, from (15), (16), (17) and the above argument, we have $\frac{dV}{dt} \leq 0$. The equality only holds when $x_r(t) = \bar{x}_r(t)$ and for each link $\lambda_l(t) = \bar{\lambda}_l(t)$ or $\bar{y}_l(t) = c_l(t)$. Consequently, we conclude that the primal-dual algorithm is asymptotically stable in trajectory-stability.

4 Stability Analysis with Delay

In the previous section, the trajectory stability of the primal-dual approach is analyzed in the absence of delay. When delay is considered, generally global stability is hard to obtain [18]. Instead, in this section, we will focus on local stability of the congestion control scheme in the presence of round trip time delay. Our focus is to obtain the sufficient conditions on system parameters for the system to be locally stable. As it is reasonable to assume that the trajectories of nonlinear systems in a small neighborhood of an equilibrium point to be "close" to the trajectories of its linearization near that point [10], we still use linearization technique to study local stability of congestion control with time delay, an approach also adopted by [11–13, 23].

Let τ_{lr}^f be the "forward" propagation delay from r's source $s(r)$ to link l and τ_{lr}^b be the "reverse" propagation delay from link l to the source $s(r)$ of route r. Then, $T_r = \tau_{lr}^f + \tau_{lr}^b$ for $l \in r$ is defined as the round trip delay (or round trip time, RTT) from source $s(r)$ to l. Therefore, with this propagation delay considered, the source rate controller becomes

$$\dot{x}_r(t) = k_r \left(w_r - x_r(t - T_r) q_r(t)\right) \tag{18}$$

where $q_r(t)$ is the route price given by

$$q_r(t) = \sum_{l\in r} \lambda_l(t - \tau_{lr}^b). \tag{19}$$

Our analysis will follow these steps. We will first linearize (18) around its local equilibrium point. Then we will transform the system to the frequency domain and employ the Nyquist stability criterion to derive sufficient conditions for

$$
\begin{aligned}
\frac{dV}{dt} &= \sum_{r\in R} \frac{1}{k_r x_r(t)}(x_r(t)-\bar{x}_r(t))\dot{x}_r(t) + \sum_{l\in L}\frac{1}{h_l(\lambda_l(t))}(\lambda_l(t)-\bar{\lambda}_l(t))\dot{\lambda}_l(t) \\
&= \sum_{r\in R}(x_r(t)-\bar{x}_r(t))\left(\frac{w_r}{x_r(t)} - \sum_{l\in r}\lambda_l(t)\right) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(y_l(t)-c_l(t))^+_{\lambda_l(t)} \\
&= \sum_{r\in R}(x_r(t)-\bar{x}_r(t))\left(\frac{w_r}{x_r(t)} - q_r(t)\right) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(y_l(t)-c_l(t))^+_{\lambda_l(t)} \\
&\le \sum_{r\in R}(x_r(t)-\bar{x}_r(t))\left(\frac{w_r}{x_r(t)} - q_r(t)\right) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(y_l(t)-c_l(t)) \\
&= \underbrace{\sum_{r\in R}(x_r(t)-\bar{x}_r(t))(\bar{q}_r(t)-q_r(t)) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(y_l(t)-\bar{y}_l(t))}_{\text{Sum 1}} \\
&\quad + \underbrace{\sum_{r\in R}(x_r(t)-\bar{x}_r(t))\left(\frac{w_r}{x_r(t)} - \bar{q}_r\right) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(\bar{y}_l(t)-c_l(t))}_{\text{Sum 2}}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\text{Sum 1} &= \sum_{r\in R}(x_r(t)-\bar{x}_r(t))(\bar{q}_r(t)-q_r(t)) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(y_l(t)-\bar{y}_l(t)) \\
&= (\bar{\mathbf{q}}(t)-\mathbf{q}(t))^T(\mathbf{x}(t)-\bar{\mathbf{x}}(t)) + (\boldsymbol{\lambda}(t)-\bar{\boldsymbol{\lambda}}(t))^T(\mathbf{y}(t)-\bar{\mathbf{y}}(t)) \\
&= (\bar{\boldsymbol{\lambda}}(t)-\boldsymbol{\lambda}(t))^T\mathcal{R}(\mathbf{x}(t)-\bar{\mathbf{x}}(t)) + (\boldsymbol{\lambda}(t)-\bar{\boldsymbol{\lambda}}(t))^T(\mathbf{y}(t)-\bar{\mathbf{y}}(t)) \\
&= (\bar{\boldsymbol{\lambda}}(t)-\boldsymbol{\lambda}(t))^T(\mathbf{y}(t)-\bar{\mathbf{y}}(t)) + (\boldsymbol{\lambda}(t)-\bar{\boldsymbol{\lambda}}(t))^T(\mathbf{y}(t)-\bar{\mathbf{y}}(t)) \\
&= 0
\end{aligned} \tag{16}
$$

$$
\text{Sum 2} = \sum_{r\in R}(x_r(t)-\bar{x}_r(t))\left(\frac{w_r}{x_r(t)} - \bar{q}_r\right) + \sum_{l\in L}(\lambda_l(t)-\bar{\lambda}_l(t))(\bar{y}_l(t)-c_l(t)) \tag{17}
$$

the system to be locally stable.

Introducing $\delta x_r(t) = x_r(t) - \bar{x}_r$ and $\delta q_r(t) = q_r(t) - \bar{q}_r$ where $\bar{x}_r$ and $\bar{q}_r$ are corresponding local equilibrium points of $x_r(t)$ and $q_r(t)$ respectively. By linearizing (18) around the equilibrium points $\bar{x}_r$ and $\bar{q}_r$, we have

$$\delta\dot{x}_r(t) = -k_r(\bar{q}_r\delta x_r(t-T_r) + \bar{x}_r\delta q_r(t)). \tag{20}$$

Taking the *Laplace* transform of the above equation and noting the fact that $\bar{q}_r = w_r/\bar{x}_r$ yields

$$\left(s + \frac{k_r w_r}{\bar{x}_r}e^{-sT_r}\right)x_r(s) = -k_r\bar{x}_r q_r(s) + x_r(0). \tag{21}$$

Let $D(s)$ be the diagonal matrix of RTTs in the Laplace domain, i.e., $D(s) = \text{diag}\{e^{-sT_r}\}$. Let $X = \text{diag}\{\bar{x}_r\}$, $W = \text{diag}\{w_r\}$ and $K = \text{diag}\{k_r\}$. The above Laplace transform equation can be rewritten as

$$\left(sI + KWX^{-1}D(s)\right)\mathbf{x}(s) + KX\mathbf{q}(s) = \mathbf{x}_0, \tag{22}$$

where $\mathbf{x}_0$ is the column vector of initial states given by $\mathbf{x}_0 = (x_{0r}, r \in R)$ and I is the identify matrix given by $I = \text{diag}\{1\}$.

For the price updating procedure depicted in (19), we take the Laplace transform and obtain

$$q_r(s) = \sum_{l\in r} e^{-s\tau^b_{lr}}\lambda_l(s) = e^{-sT_r}\sum_{l\in r} e^{s\tau^f_{lr}}\lambda_l(s) \tag{23}$$

Let $\mathcal{R}(s)$ denote $|L| \times |\mathcal{S}|$ Laplace domain routing matrix that includes both routing and delay information, whose (l, r) entry is defined as

$$\mathcal{R}_{lr} = \begin{cases} e^{-s\tau^f_{lr}} & \text{if } l \in r, \\ 0 & \text{otherwise.}\end{cases} \tag{24}$$

Thus, from (23), we have

$$\mathbf{q}(s) = D(s)\mathcal{R}^T(-s)\boldsymbol{\lambda}(s) \tag{25}$$

where $\mathbf{q}(s) = (q_r(s), r \in R)$, $\boldsymbol{\lambda}(s) = (\lambda_l(s), l \in L)$.

Now we need to compute $\lambda_l(s)$ based on the price updating algorithm given in Equation (13). In order to keep mathematical tractability, instead of taking the Laplace transform directly over $\lambda_l(t)$, we first assume that at equilibrium status, $c_l(t) = \bar{c}_l$ is a constant and introduce perturbations around $\bar{c}_l$ to "emulate" the effects of a time varying $c_l(t)$. By investigating the consequences of perturbation on the source controller and link dynamics, the effects of time varying equilibrium point can be obtained. This technique has been well applied in the control domain and proven to be effective [10].

Following this approach, for the link price updating procedure given in (13), we introduce $\delta\lambda_l(t) = \lambda_l(t) - \bar{\lambda}_l$, $\delta z_l(t) = c_l(t) - \bar{c}_l$, and $\delta y_l(t) = y_l(t) - \bar{y}_l$. Linearizing (13) around equilibrium $\bar{\lambda}_l$ gives

$$\delta\dot{\lambda}_l(t) = h_l(\bar{\lambda}_l)[\delta y_l(t) - \delta z_l(t)] \tag{26}$$

Taking Laplace transform for the above equation, we have

$$\lambda_l(s) = \frac{1}{s}h_l(\bar{\lambda}_l)[y_l(s) - z_l(s)]. \tag{27}$$

As a result, we have

$$\boldsymbol{\lambda}(s) = \frac{1}{s}\text{diag}\left\{h_l(\bar{\lambda}_l)\right\}[\mathbf{y}(s) - \mathbf{z}(s)], \tag{28}$$

where $\mathbf{y}(s)$ and $\mathbf{z}(s)$ are column vectors and $\mathbf{y}(s) = (y_l(s), l \in L)$, $\mathbf{z}(s) = (z_l(s), l \in L)$.

In order to continue our discussion, a specific form of $h_l(\bar{\lambda}_l)$ is needed. For this purpose, we choose $h_l(\bar{\lambda}_l) = \beta_l\bar{\lambda}_l/\bar{c}_l$ as in [23]. Here β is often termed damping factor. In the remainder of this paper, we will employ this function for price adjustment at the links. Based on this, from equations (25) and (28), we have

$$\mathbf{q}(s) = \frac{1}{s}D(s)\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}[\mathbf{y}(s) - \mathbf{z}(s)]. \tag{29}$$

Hence,

$$\left[sI + KWX^{-1}D(s) + \frac{1}{s}KXD(s)\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathcal{R}(s)\right]X(s) = \mathcal{Z}(s) \tag{30}$$

where $\mathcal{Z}(s) = X_0 + (1/s)KXD(s)\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathbf{z}(s)$.

We remark that Equation (30) is different from corresponding equations in [23] in that, in (30), the perturbation of link capacity results in the existence of the second term in $\mathcal{Z}(s)$. This term depicts the effect of capacity variations on the system and for the purpose of stability analysis must be investigated.

From control theory, the system described by (30) is stable if all its poles lie in the left-half of the complex plane. In other words, the solutions to

$$\det(sI + Q(s)) = 0 \tag{31}$$

should only have negative real parts, where

$$Q(s) = KWX^{-1}D(s) + \frac{1}{s}KXD(s)\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathcal{R}(s) \tag{32}$$

We now will determine the conditions for the system to satisfy this requirement. Let $G(s) = (1/s)Q(s)$, then we can write

$$G(s) = \frac{1}{s}\left[KWX^{-1}D(s) + \frac{1}{s}KXD(s)\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathcal{R}(s)\right]. \tag{33}$$

Or equivalently,

$$G(s) = \text{diag}\left\{\frac{e^{-sT_r}}{sT_r}\right\}\text{diag}\{k_rT_r\}X\left[WX^{-2} + \frac{1}{s}\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathcal{R}(s)\right]. \tag{34}$$

From the generalized Nyquist Criterion [3], the stability condition of the proposed congestion control scheme is equivalent to the following statement: the eigenvalues of $G(j\omega)$ should not encircle the point -1. Therefore to guarantee local stability, we have to find conditions when the eigenvalues of $G(j\omega)$ do not encircle -1 for all values of ω. We will closely follow the line of analysis in [11]. From *Lemma* 3.1 in [23], we know that there exists an ω^* such that no eigenvalues of $G(j\omega)$ is real for all $\omega < \omega^*$. Therefore, we only need to prove that under some constraints, the eigenvalues of $G(j\omega)$ don't enclose -1 for $\omega > \omega^*$.

For ease of exposition, we define

$$G_1 = \text{diag}\left\{\sqrt{k_rT_r\bar{x}_r}\right\}WX^{-2}\text{diag}\left\{\sqrt{k_rT_r\bar{x}_r}\right\}$$

$$G_2 = \text{diag}\left\{\sqrt{k_rT_r\bar{x}_r}\right\}\mathcal{R}^T(-j\omega)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathcal{R}(j\omega)\cdot \text{diag}\left\{\sqrt{k_rT_r\bar{x}_r}\right\}$$

$$L = \text{diag}\left\{\frac{e^{-j\omega T_r}}{j\omega T_r}\right\}.$$

Therefore,

$$G(j\omega) = \left(G_1 + \frac{1}{j\omega}G_2\right)L \tag{35}$$

In the following, we will derive certain conditions under which the eigenvalues of G_1L do not encircle $-\varepsilon$ and conditions for the eigenvalues of $\frac{1}{j\omega}G_2L$ to be bounded by $(1-\varepsilon)$. Hence, if the conditions are jointly satisfied, the eigenvalues of $G(j\omega)$ will not encircle -1.

Note that

$$\sigma(G(j\omega)) = \sigma\left(\left(G_1 + \frac{1}{j\omega}G_2\right)L\right)$$

where $\sigma(\cdot)$ denotes the spectrum of a square matrix. Let λ be an eigenvalue of $G(j\omega)$ and $\mathbf{v}$ be the corresponding normalized eigenvector, i.e. $\|\mathbf{v}\|^2 = \mathbf{v}^*\mathbf{v} = 1$. Note that G_1 is a positive definite matrix and G_2 is a Hermitian matrix. According to the matrix theory, we have

$$\lambda\mathbf{v} = \left(G_1 + \frac{1}{j\omega}G_2\right)L\mathbf{v} \tag{36}$$

As G_1 is positive definite, the inverse G_1^{-1} exists. Therefore, from (36) we have

$$\lambda\mathbf{v}^*G_1^{-1}\mathbf{v} = \mathbf{v}^*\left(I + \frac{1}{j\omega}G_1^{-1}G_2\right)L\mathbf{v}. \tag{37}$$

Hence,

$$\lambda = \frac{\mathbf{v}^*L\mathbf{v}}{\mathbf{v}^*G_1^{-1}\mathbf{v}} + \frac{1}{j\omega}\frac{\mathbf{v}^*G_1^{-1}G_2L\mathbf{v}}{\mathbf{v}^*G_1^{-1}\mathbf{v}}. \tag{38}$$

For the first term in (38), we note that

$$\mathbf{v}^*L\mathbf{v} = \sum_r |v_r|^2\frac{e^{-j\omega T_r}}{j\omega T_r},$$

and

$$\mathbf{v}^*G_1^{-1}\mathbf{v} \geq \lambda_{\min}(G_1^{-1}) = \frac{1}{\lambda_{\max}(G_1)}.$$

Thus,

$$\sigma(G_1L) \subset \lambda_{\max}(G_1)\sum_r |v_r|^2\frac{e^{-j\omega T_r}}{j\omega T_r} \tag{39}$$

Therefore, based on the fact that as ω is varied from $-\infty$ to ∞, $\frac{\pi}{2}\frac{e^{-j\omega T_r}}{j\omega T_r}$ does not encircle the point -1(see [23] for its proof), we can choose k_r such that $\lambda_{\max}(G_1) < \varepsilon\frac{\pi}{2}$ and consequently force $\sigma(G_1L)$ not to encircle $-\varepsilon$, $\forall\, 0 < \varepsilon < 1$. Specifically, the condition to satisfy the above requirement for G_1 is given by

$$k_r\bar{q}_r < \varepsilon\frac{\pi}{2T_r} \quad \forall r \in R. \tag{40}$$

Next, we will analyze the second term in (38). Our goal is to prove

$$\left|\frac{1}{j\omega}\frac{\mathbf{v}^*G_1^{-1}G_2L\mathbf{v}}{\mathbf{v}^*G_1^{-1}\mathbf{v}}\right| < 1 - \varepsilon$$

Note that

$$\left|\frac{1}{j\omega}\frac{\mathbf{v}^*G_1^{-1}G_2L\mathbf{v}}{\mathbf{v}^*G_1^{-1}\mathbf{v}}\right| \leq \frac{1}{\omega^*}\frac{\|G_1^{-1}\|_2\|G_2\|_2\|L\|_2}{\lambda_{\min}(G_1^{-1})} \tag{41}$$

where $\|\cdot\|_2$ is the matrix norm given by $\|A\|_2 = \sqrt{\lambda_{\max}(A^*A)}$ [22]. The above inequality follows directly from the Cauchy-Schwartz inequality and $\lambda_{\min}(A) \leq \mathbf{x}^TA\mathbf{x} \leq \lambda_{\max}(A)$ for any vector $\mathbf{x}$ satisfying $\mathbf{x}^T\mathbf{x} = 1$ and any positive-definite matrix A.

Now, we investigate the RHS of inequality (41)

$$\begin{aligned}\frac{1}{\omega^*}&\frac{\|G_1^{-1}\|_2\|G_2\|_2\|L\|_2}{\lambda_{\min}(G_1^{-1})}\\ &\leq \frac{1}{(\omega^*)^2}\max_r\left(\frac{1}{T_r}\right) \times \frac{\lambda_{\max}(G_1)\lambda_{\max}(G_2)}{\lambda_{\min}(G_1)}\\ &\leq \frac{1}{(\omega^*)^2}\max_r\left(\frac{1}{T_r}\right)\lambda_{\max}(G_2)\end{aligned} \tag{42}$$

Let $\rho(\cdot)$ denote spectral radius of a square matrix, and $\hat{\mathcal{R}}(j\omega) = \text{diag}\left\{\sqrt{\beta_l\bar{\lambda}_l/\bar{c}_l}\right\}\mathcal{R}(j\omega)\text{diag}\left\{\sqrt{k_rT_r\bar{x}_r}\right\}$, then we have

$$\begin{aligned}\lambda_{\max}(G_2) &= \rho\left(\hat{\mathcal{R}}^T(-j\omega)\hat{\mathcal{R}}(j\omega)\right)\\ &= \rho(\text{diag}\{k_rT_r\bar{x}_r\}\mathcal{R}^T(-j\omega)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathcal{R}(j\omega))\\ &\leq \|\text{diag}\{k_rT_r\}\mathcal{R}^T(-j\omega)\text{diag}\{\beta_l\bar{\lambda}_l\}\|\\ &\quad\times\|\text{diag}\left\{\frac{1}{\bar{c}_l}\right\}\mathcal{R}(j\omega)\text{diag}\{\bar{x}_r\}\|\\ &\leq \|\text{diag}\{k_rT_r\}\mathcal{R}^T(-j\omega)\text{diag}\{\beta_l\bar{\lambda}_l\}\| \times 1\end{aligned} \tag{43}$$

The last inequality above uses the fact that at equilibrium point

$$\sum_{r:l\in r}\bar{x}_r = \bar{y}_l \leq \bar{c}_l, \quad \forall l \in L.$$

For the first term in the last inequality in (43), if we set

$$\beta_l < \frac{\min_r(T_r)}{k_rT_r\bar{q}_r}(\omega^*)^2 \times (1-\varepsilon), \tag{44}$$

we have

$$\|\text{diag}\{k_rT_r\}\mathcal{R}^T(-j\omega)\text{diag}\{\beta_l\bar{\lambda}_l\}\| \leq \min_r(T_r)(\omega^*)^2(1-\varepsilon).$$

Therefore,

$$\lambda_{\max}(G_2) < \min_r(T_r)(\omega^*)^2(1-\varepsilon) \tag{45}$$

Combining (42) and (45), we have

$$\frac{1}{(\omega^*)}\frac{\|G_1^{-1}\|_2\|G_2\|_2\|L\|_2}{\lambda_{\min}(G_1^{-1})} < 1 - \varepsilon \tag{46}$$

Till now, we have proven that the first term of (38) does not encircle ε given the condition in (40) is satisfied and the second term (38) does not encircle $(1-\varepsilon)$ as shown in (46) and (41) as long as (44) is satisfied. Therefore, for any $\omega > \omega^*$, if the two conditions are satisfied, the eigenvalues of $G(j\omega)$ will not encircle -1 and hence the system is locally stable.

Therefore, the sufficient conditions for guaranteeing the system with time delay to be locally stable can be summarized as

$$\begin{cases}k_r\bar{q}_r \leq \varepsilon\frac{\pi}{2T_r}, & \forall\, r \in R \quad \text{and}\\ \beta_l < \frac{\min_r(T_r)}{k_rT_r\bar{q}_r}(\omega^*)^2 \times (1-\varepsilon), & \forall\, r \in R:\ l \in r\end{cases} \tag{47}$$

5 Sensitivity Analysis of Link Capacity with Perturbation

In this section, we will study how to reduce the effects owing to link capacity perturbation. Equivalently, our goal is to study proper system parameters so that the system is robust to the perturbations. In other words, if the system is insensitive to capacity perturbations, rate oscillations will be kept at a low level even in the presence large capacity change. Such a feature is much desired by wireless networks for example, in order to provide quality of service to higher layer applications.

Before going to the details, let us first refresh system sensitivity using a system characterized by linear equation $Ax = b$. For this example system, when A and b are subjected to small order perturbation ΔA and Δb respectively, the problem becomes $(A + \Delta A)(x + \Delta x) = b + \Delta b$. Our main concern is the deviation Δx of the solution with respect to the perturbation of ΔA and Δb. The system sensitivity is thus defined as the extent of the deviation of Δx relative to ΔA and Δb [2]. Putting this into the congestion control scheme we are concerned, our main target is to investigate the deviation of system equilibrium relative to the link capacity change.

Let us consider Equation (30). We remark that as X_0 is the initial condition vector, its existence will not affect our sensitivity analysis and hence can be ignored. Towards this end, we define

$$\mathcal{Z}'(s) = (1/s)KXD(s)\mathcal{R}^T(-s)\text{diag}\left\{\frac{\beta_l\bar{\lambda}_l}{\bar{c}_l}\right\}\mathbf{z}(s), \quad (48)$$

and our focus becomes

$$[sI + Q(s)]X(s) = \mathcal{Z}'(s). \quad (49)$$

General sensitivity analysis will involve matrices (in Laplace domain) $\mathcal{R}(s)$, $\mathcal{R}^T(-s)$, and $[sI+Q(s)]^{-1}$. Computing of $[sI + Q(s)]^{-1}$ is often dependent on the specific system setup and become intractable for complex systems. To illustrate the idea and avoid tedious matrix manipulations, we consider a simple scenario of one source transmitting over one link in the following discussion. Then (49) can be expressed as

$$\left(s + k\bar{\lambda}e^{-sT} + \frac{1}{s}\frac{k\beta\bar{\lambda}\bar{x}e^{-sT}}{\bar{c}}\right)x(s) = \frac{1}{\bar{c}s}k\beta\bar{\lambda}\bar{x}e^{-sT}e^{s\tau^f}z \quad (50)$$

At equilibrium, $\bar{c} = \bar{x}$ and $\bar{\lambda} = w/\bar{x}$. From the above equation, we have

$$\left(s + \frac{kw}{\bar{c}}e^{-sT} + \frac{1}{s}\frac{k\beta we^{-sT}}{\bar{c}}\right)x(s) = \frac{k\beta w}{\bar{c}s}e^{-sT}e^{s\tau^f}z \quad (51)$$

and it follows that

$$x(s) = \frac{k\beta we^{-sT}e^{s\tau^f}}{\bar{c}s^2 + kwe^{-sT}s + k\beta we^{-sT}}z. \quad (52)$$

Define

$$T(s) = \frac{k\beta we^{-sT}e^{s\tau^f}}{\bar{c}s^2 + kwe^{-sT}s + k\beta we^{-sT}}. \quad (53)$$

In order to minimize the effect of link perturbation on source rate, we need to minimize $|T(s)|^2_{s=j\omega}$, that is,

$$\min_{k,\beta} \quad |T(s)|^2_{s=j\omega} = \left|\frac{k\beta w}{\bar{c}s^2 + kwe^{-sT}s + k\beta we^{-sT}}\right|^2_{s=j\omega}. \quad (54)$$

Evidently, the above optimization depends on delay T, the source controller gain k, and link damping factor β. Given the time delay T on the link, the range of k and β for guaranteeing the system local stability can be obtained through Inequality (40) and (44). Constrained thereby, the above optimization problem can then be solved to achieve the optimal balancing tradeoff between system stability and sensitivity. Often, closed form expression is hard to obtain and we will rely on numerical study in section 6 for further investigation.

6 Experimental Study

In this section, we perform a broad set of experiments to validate our theoretical results and study the effects of different control parameters, for example, source controller gain k and damping factor β.

6.1 System Setup

The dual algorithm at a link for price updating in (13) is implemented by marking packets with probability (the link price) λ_l as an exponential function of the queue length b_l, given by

$$\lambda_l = \begin{cases} 0 & \text{if} \quad 0 \le b_l < th_{\min,l}, \\ p_{\min}e^{\frac{\beta_l}{c_l}(b_l - th_{\min,l})} & \text{if} \quad th_{\min,l} < b_l < th_{\max,l}, \\ 1 & \text{if} \quad b_l \ge th_{\max,l} \end{cases} \quad (55)$$

where $th_{\min,l} < th_{\max,l}$ are the two user defined queue length thresholds and $p_{\min}$ is the marking probability when $b_l = th_{\min,l}$. In our simulation, $th_{\min} = 4$, $th_{\max} = 10$ packets, and $p_{\min} = 0.002$. This scheme is termed E-RED and details are given by [11]. Again, the primal algorithm to perform rate updating is given by

$$\dot{x}_r(t) = k_r\left(w_r - x_r(t)\sum_{l\in r}\lambda_l\right). \quad (56)$$

The simulations are carried out in *NS*-2 platform and the network topology is depicted in Fig. 1. In this topology, there are totally four flows and R_1R_2 is the interested bottleneck link whose capacity is originally set to be 1 Mbps. Capacity variations are simulated by adding a sine wave with a period of 60 seconds and different amplitudes to the fixed capacity. An extensive set of other perturbations have also been simulated which demonstrate similar results. This one is chosen for ease of explanation and limit of space. For all other links, the capacities are fixed to be 5Mbps. The buffer limit of the bottleneck link is set to be 500 packets and all packets are fixed to 512 bytes.

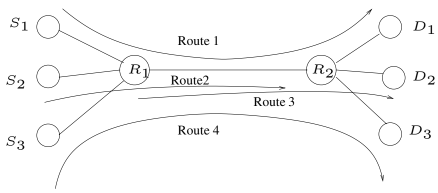
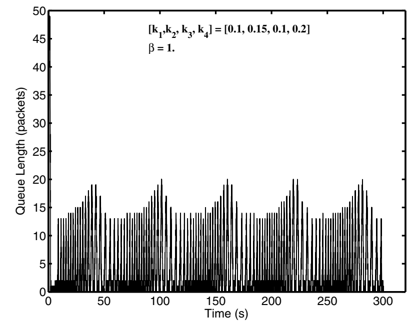


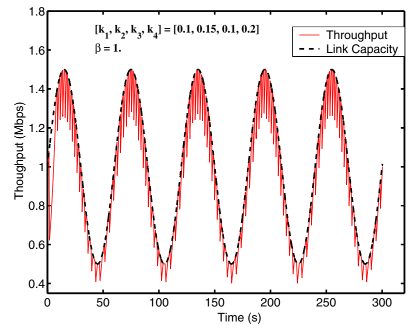
Figure 1. Simulated Network topology

6.2 Global Stability

For fixed link capacities, extensive results have been shown in the literature on global stability in the absence of delay, and hence we focus on scenarios with time varying capacities. The results are depicted in Fig. 2(a) and (b). From the figures, we can see that both the link utilization and queue length is trajectory stable in the sense that it can closely follow the capacity oscillation after short transient phases. Through our simulations, we further notice that this stability is actually not affected by different parameters, namely the source controller gain k and damping factor β.



(a) Queue length at the bottleneck link



(b) Utilization of the bottleneck link

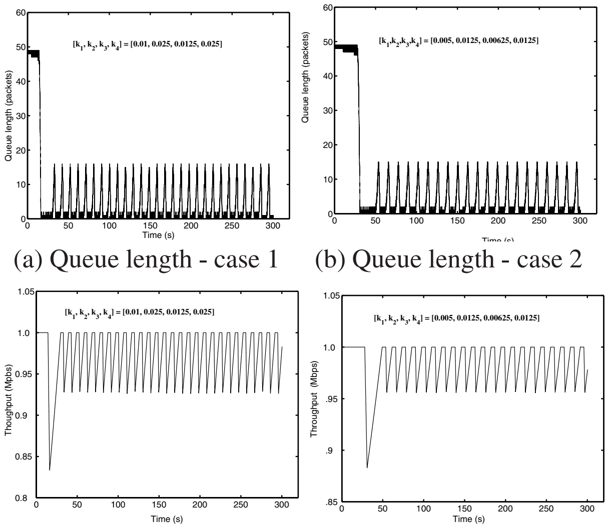
Figure 2. Global Stability with link perturbation

6.3 Local Stability and Sensitivity

However, as shown in our analysis, the local stability when delay is present significantly depends on control parameters such as k, β, and RTTs of the system. In this set of simulations, we first study the cases when link capacity of the bottleneck is fixed. We adjust the source controller gain while keeping other parameters fixed. We first set source controller gains in the system as $[k_1\ k_2\ k_3\ k_4] = [0.01\ 0.025\ 0.0125\ 0.025]$ where k_i is the gain for source i. Then we reduce all the gains by half and study their differences. The results for these two cases are shown in Fig. 3. From Fig. 3(a) and (b), we can see that the queue length is being kept between 0 and 15 packets after the transient phase, denoting a stable condition. Correspondingly, Fig. 3(c) and (d) show that the throughput of the bottleneck link oscillates around the equilibriums slightly. By comparing these two cases with different controller gains, we observe that adjusting source controller gains can affect the throughput oscillation magnitude around the equilibrium. In the simulations, we also find that our algorithm is not sensitive to the propagation time delay and damping factor β for price updating at the link.

In the presence of capacity variation and system delay, we observe in our simulation that if perturbation is significant (large amplitude of the sine wave), the system is unable to stabilize regardless of the parameters chosen. This actually concurs with our analytical results as given a certain perturbation, the sufficient conditions may not be able to be satisfied by adjusting the system parameters. For a reasonable perturbation, however, suitable parameters chosen according to the derived conditions can stabilize the system. A typical stable scenario with the amplitude of the sine wave set to be 0.2Mbps is shown in Fig. 4. We can see that the system can effectively adapt to the capacity change and achieve stabilized queue length and link utilization. We also observe in our simulation that the throughput of the bottleneck link has different degrees of sensitivity to different values of k while maintaining this local stability.

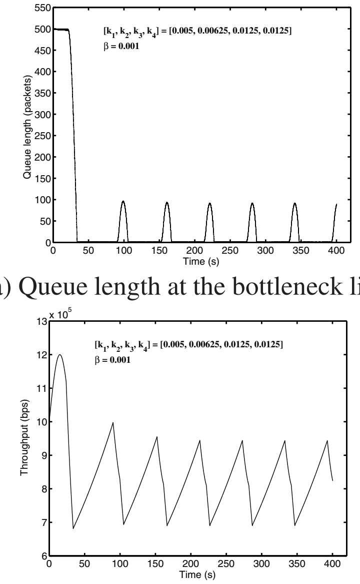
On the contrary, if the parameters are not chosen properly according to the conditions derived, the system will not be able to stabilize either. For a large source controller gain, the results are depicted in Fig. 5(a) and (b), where the large oscillations for the link utilization reveal an unstable system. Similar results are observed for unsatisfactory values of the damping gain and different RTTs which are not shown due to space limit.



(a) Queue length - case 1 (b) Queue length - case 2

(c) Utilization - case 1 (d) Utilization - case 2

Figure 3. Local Stability without link perturbation



(a) Queue length at the bottleneck link

(b) Utilization of the bottleneck link

Figure 4. Local Stability with link perturbation - stable scenario

7 Related Work

Congestion control in wireline domain has attracted tremendous research interests owing the inadequacy of TCP when facing high bandwidth-delay product. While extensive results exist [18, 23], a common assumption is constant link capacity. Obviously, this assumption dictates the problem and results presented therein deviate from our focus in this paper, namely the time varying capacity.

Alongside, various Active Queue Management (AQM) schemes have been designed for congestion detection at intermediate routers and provide indication to the sources. Notably among them are random early detection (RED), its numerous variants, proportional integer (PI), random exponential marking (REM), just to name a few [12]. In particular, an Adaptive Virtual Queue(AVQ) algorithm is proposed in [12]. Although AVQ scheme adaptively changes the virtual link capacity to get high utilization via predefined dynamic law, its dynamic behavior is different from that owing to time-varying wireless channels. The time varying link capacity in wireless networks cannot be described by a deterministic differential equation and hence the research results for AVQ cannot be directly applied here.

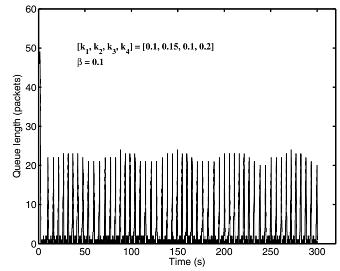
In the wireless domain, recent research in designing congestion control protocols has adopted the distributed control framework proposed by Kelly. It is pointed out in [5] that variable bandwidth is one of the intrinsic characteristics that affect the performance of transport protocols in wireless networks. However, the authors did not address how bandwidth variations affect the system performance, nor was any formal analysis regarding this presented.

A distributed hop-by-hop congestion control scheme is developed in [24] for multi-hop wireless networks. The scheme is shown to be stable in the absence of round propagation delay. However, the authors assume that channel variations can be effectively masked by the physical layer coding and modulation schemes and hence can be considered as a “constant channel” at higher layers.

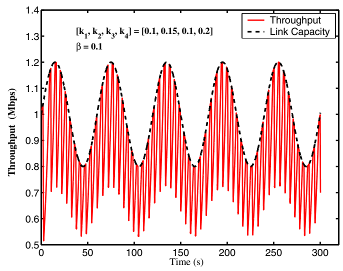
Joint congestion control and power control in wireless multi-hop networks is investigated in [1]. As larger transmission power can increase channel capacity in wireless networks, the authors thus propose to allocate proper amount of power to bottleneck links in order to elevate the congestion condition and hence increase system wide utility. The key challenge lies in the interference among multiple links, as power increase on one link will simultaneously degrade other links’ signal-to-noise ratio and hence capacity. The author assumes that this interference information is know globally, which essentially limits its application scope.

8 Conclusion and Future Work

By explicitly introducing time varying channel capacity into the utility maximization problem, we investigate congestion control in multi-hop wireless networks following Kelly’s seminal work. Different from conventional system stability around an equilibrium point, we employ trajec-



(a) Queue length at the bottleneck link



(b) Utilization of the bottleneck link

Figure 5. Local Stability with link perturbation - unstable scenario

tory stability and prove the proposed prime-dual algorithm is stable around a time varying trajectory without considering system delay. In the presence of system delay, we derive sufficient conditions for the system to be locally stable. Furthermore, by modeling link variations as perturbations to constant capacities, we investigate system sensitivity and provide insightful experimental studies regarding its tradeoff with system stability.

Our ongoing efforts is simulation of large scale multihop wireless networks and further investigation of system sensitivity in more general system setups.

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